Maths

(Pages : 4) **U – 2376**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1541 — REAL ANALYSIS — I

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Let $f(x)=x^2$. If $A=[0,2]$, $B=[1,4]$, find $f(A)$ and $f(B)$.

2. Show that $|ab|=|a|\,|b|$ for all $a, b\in R$.

3. Give an example for a one-one function from $(-1,1)$ onto R.

4. Find infimum of the set $\left\{2+\frac{3}{n};\ n\in N\right\}$.

5. Write the first five terms of the sequence defined inductively by $x_1=2, x_{n+1}=\frac{x_n+1}{2}$.

P.T.O.

6. Find $\lim\left(\frac{2}{5}\right)^n$.

7. Give an example for a monotone sequence that is not Cauchy.

8. Check whether $(1,5)$ is compact.

9. State true or false: Union of two connected sets is connected. Justify your answer.

10. Define nowhere dense set and give example.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry 2 marks each.

11. Show that the set $E=\{2n;\, n \in N\}$ is countable.

12. Show that $\sqrt[3]{2}$ is algebraic.

13. Show that the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \ldots\right)$ is divergent.

14. Give an example of a series which is convergent but not absolutely convergent.

15. Show that the sequence $\left(\frac{1}{n}\right)$ is Cauchy.

16. Show that every convergent sequence is bounded.

17. Show that $\lim(a_n + b_n) = \lim a_n + \lim b_n$.

18. Give an example for an unbounded sequence which contain a subsequence that is Cauchy.

19. If $\sum_{k=1}^{\infty} a_k = A$, show that $\sum_{k=1}^{\infty} ca_k = cA$.

20. Show that $A = \left\{\frac{1}{n};\, n \in N\right\}$ is not closed.

21. For any $A \subseteq R$, show that the closure A is the smallest closed set containing A.

22. Give an example of a disconnected set whose closure is connected.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. State and prove Nested Interval property.

24. Given any number $x \in R$, show that there exist $n \in N$ satisfying $n > x$.

25. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

26. Construct a sequence that converges to $\sqrt{2}$.

27. Show that two real numbers a, b are equal if and only if for every real number $\varepsilon > 0$, $|a - b| < \varepsilon$.

28. Define geometric series. Discuss its convergence.

29. Show that if a set $K \subseteq R$ is compact. then it is closed and bounded.

30. Show that a point x is a limit point of a set A if and only if $x = \lim a_n$ for some sequence (a_n) contained in A satisfying $a_n \neq x$ for all $n \in N$.

31. Construct an open cover for (0,1) in such a way that it has no finite sub cover.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) Show that the set Q is countable.

    (b) State and prove Canto's theorem.

33. (a) State and prove Cauchy condensation test.

    (b) Let $Y=(y_n)$ be defined inductively by $y_1=1, y_{n+1}=\frac{1}{4}(2y_n+3)$ for $n\geq 1$. Find $\lim Y$.

34. (a) Show that a sequence converges if and only if it is a Cauchy sequence.

    (b) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2^n}$.

35. Show that a non empty perfect set is uncountable.

(2 × 15 = 30 Marks)

(Pages : 4) **U – 2377**

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First Degree Programme under CBCSS

Mathematics

Core Course

MM 1542 : COMPLEX ANALYSIS – I

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find i^{62}.
2. Describe the set of points Rez $\geq$ 4.
3. Find Arg10i.
4. State DeMoivre's formula.
5. Find boundary of $0 < |z-2| < 3$.
6. Define analytic function in a domain.
7. State Morera's theorem.
8. Define sin z.
9. Define complex exponent z^{α}, where $z \neq 0$ and α is a complex constant.
10. Evaluate $\int_0^1 (2t + it^2)dt$.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. Write the number $((3-i)^2-3)i$ in the form $a+bi$.

12. Write $(\sqrt{3}-i)^2$ in polar form.

13. Find $(1+i)^{24}$.

14. Show that $|e^z| \leq 1$, if $Rcz \leq 0$.

15. Find points at which $f(z)=\dfrac{iz^3+2z}{z^2+1}$ is not analytic.

16. Prove that e^{iz} is periodic with a period 2π.

17. Find all poles and their multiplicities of the function $f(z)=\dfrac{z^2+1}{(z-2)(z-3)^4}$.

18. Describe analyticity of log z.

19. Find the Taylor form of the polynomial $g(z)=(z-1)(z-2)^3$ centred at $z=2$.

20. Define simply connected domain. Give an example.

21. Find $\int_C \dfrac{\cos z}{z^2+z-12}dz$, where C is $|z|=2$.

22. Compute $\int_C \cos z dz$, where C is the contour formed by upper semi circle of $|z|=1$ from –1 to 1 followed by line segment from 1 to $2+i$. (Use independence of path).

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Find all values of $(-16)^{\frac{1}{4}}$.

24. Write $f(z)=\dfrac{2z^2+3}{|z-1|}$ in $u(x, y)+iv(x, y)$ form.

25. Prove that $f(z) = e^z$ is entire and find its derivative.

26. Show that if f is analytic in a domain D and either $\operatorname{Re}(f(z))$ or $\operatorname{Im}(f(z))$ is constant, then $f(z)$ must be constant.

27. A polynomial $p(z)$ of degree 4 has zeros at the points -1, $3i$ and $-3i$ of respective multiplicities 2, 1 and 1. If $p(1)=80$, find $p(z)$.

28. Find all values of $(-2)^i$.

29. Show that if $z_1 = i$ and $z_2 = i - 1$, then $\log(z_1 z_2) \neq \log z_1 + \log z_2$.

30. If γ is the vertical line segment from $z = R (R > 0)$ to $z = R + 2\pi i$, then show that

$$\left| \int_\gamma \frac{e^{3z}}{1+e^z} dz \right| \leq \frac{2\pi e^{3R}}{e^R - 1}.$$

31. Compute $\int_\Gamma \bar{z} dz$, where (a) Γ is in the circle $|z| = 2$ traversed once counterclockwise (b) Γ is the circle $|z|=2$ traversed once clockwise.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. (a) Evaluate $\int_0^{2\pi} \sin^4\theta \, d\theta$.

(b) State and prove the necessary conditions (Cauchy-Riemann equations) for a function to be analytic at a point.

33. Let C be the perimeter of the square with vertices at the points $z = 0$, $z = 1$, $z = 1 + i$ and $z = i$ traversed once in that order. Show that $\int_C e^z dz = 0$.

34. (a) State Cauchy's integral formula.

(b) Let C be the circle $|z| = 2$ traversed once in the positive sense. Compute each of the following integrals.

(i) $\int_C \frac{\cos z}{z^3 + 9z} dz$

(ii) $\int_C \frac{\sin z}{z^2(z-4)} dz$

(iii) $\int_C \frac{5z^2 + 2z + 1}{(z-i)^3} dz$.

35. State and prove fundamental theorem of Algebra.

(2 × 15 = 30 Marks)

(Pages : 4) **U – 2378**

Reg. No. : ....

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Fifth Semester B.Sc. Degree Examination, December 2024

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1543 : ABSTRACT ALGEBRA – GROUP THEORY

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **1** mark each.

1. Check whether the set {0,1, 2, 3} is a group under multiplication modulo 4.

2. In a group G. show that there is only one identity element.

3. Find the subgroup of Z_{30} of order 10.

4. Express (1 2 3 4 5) as product of 2-cycles.

5. Check whether the mapping $\varphi:(R,+)\to(R,+)$ defined by $\varphi(x)=x^3$ is an isomorphism.

6. Find an automorphism of the group of complex numbers under addition.

7. Let $H=\{0, \pm 3, \pm 6, \pm 9, \ldots \ldots \ldots \ldots\}$. Find all left cosets of H in Z.

P.T.O.

8. Show that all groups of order 25 is Abelian.

9. Let $\varphi : R^* \to R^*$ be defined by $\varphi(x) = |x|$. Find $Ker\varphi$.

10. How many Abelian groups (upto isomorphism) are there of order 15.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each .

11. Let G be an Abelian group. Show that $H = \{x \in G : |x| \textit{ is finite}\}$ is a subgroup of G.

12. For group elements a, b, show that $(ab)^{-1} = b^{-1}a^{-1}$.

13. Find all generators of the subgroup of order 9 in Z_{36}.

14. Show that S_3 is a non-Abelian group.

15. What is the order of the permutation $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}$.

16. Show that there is no isomorphism from Q, the group of rational numbers under addition to Q^* the group of non zero rational numbers under multiplication.

17. Show that every group of prime order is cyclic.

18. Show that a group of order 75 can have atmost one subgroup of order 25.

19. Show that the group $SL(2, R)$ is a normal subgroup of $GL(2, R)$.

20. Let φ be a homomorphism from a group G to a group G'. Show that $\varphi(a) = \varphi(b)$ if and only if $a\,Ker\varphi = b\,Ker\varphi$.

21. Show that $Z/\langle n \rangle \approx Z_n$.

22. Show that center of a group G is a subgroup of G.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. Let a be an element of order n in a group and let k be positive integer. Show that $\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$ and $|a^k| = n / \gcd(n,k)$.

24. Explain the dihedral group D_n of order $2n$.

25. Determine the number of elements in S_7 of order 12.

26. Compute $Aut(Z_{10})$.

27. Let φ be a homomorphism from a group G onto a group G'. Prove that $G = \langle a \rangle$ if and only if $G' = \langle \varphi(a) \rangle$.

28. For any two finite subgroup H and K, show that $|HK| = |H||K| / |H \cap K|$.

29. Let G be a group and let H be a normal subgroup of G. Show that the set $G/H = \{aH : a \in G\}$ is a group under the operation $(aH)(bH) = abH$.

30. Let G be a group and let $Z(G)$ be the center of G. If $G/Z(G)$ is cyclic, show that G is Abelian.

31. Show that a group of order 35 is cyclic.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) Prove that if a is the only element of order 2 in a group, then a lies in the center of the group.

    (b) Let G be a group and let $a \in G$. If a has infinite order. then show that $a^i = a^j$ if and only if $i = j$. If a has finite order n, then show that $\langle a \rangle = \{e, a, a^2, \ldots a^{n-1}\}$ and $a^i = a^j$ if and only if n divides $i - j$.

33. (a) If $\varepsilon = \beta_1\beta_2......\beta_r$, where β's are 2-cycles, then show that r is even.

(b) Show that the group of rotations of a cube is isomorphic to S_4.

34. If a group G is the internal direct product of a finite number of subgroups $H_1, H_2,......H_n$, then show that G is isomorphic to the external direct product of $H_1, H_2,......H_n$.

35. (a) Let G be a finite Abelian group of order $p^n m$, where p is a prime that does not divide m. Show that $G = H \times K$, where $H = \{X \in G : x^{p^n} = e\}$ and $K = \{x \in G : x^m = e\}$.

(b) State and prove first isomorphism theorem.

(2 × 15 = 30 Marks)

(Pages : 4) **U – 2379**

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First Degree Programme under CBCSS

Mathematics

Core Course

MM 1544 : DIFFERENTIAL EQUATIONS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the questions.

1. Find the degree of the differential equation : $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 6y + 10 = 0$.

2. Solve $y' = \frac{y}{x}$.

3. Define exact equations.

4. Find an integrating factor of the differential equation : $x\frac{dy}{dx} + 2y = 3$.

5. Find the Wronskian of $y'' + 4y = 0$.

6. Show that $y = 1 + \sin x$ is a solution of $y'' + y = 1$.

P.T.O.

7. Define standard form of Bernoulli's equation.

8. Solve $y'' + 2y' + 3y = 0$.

9. Form the differential equation of family of circles with center at origin and radius a.

10. Define singular solution of a differential equation.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions.

11. Show the linear independence of $x^2, x^2 \ln x$ by using the Wronskian.

12. Find a particular solution of $y'' + 3y' + 2y = 5x^2$.

13. Write the auxiliary equation of the Euler-Cauchy equation $x^2y'' - 5xy' + 9y = 0$.

14. Form the differential equation whose general solution is $y = cx + c - c^3$.

15. Solve $\frac{dy}{dx} - y = e^x y^2$.

16. If exact, then solve the differential equation $(2x-1)dx + (3y+7)dy = 0$.

17. Find the orthogonal trajectories of $y = mx$.

18. Solve the initial value problem $y'' + y' - 2y = 0$, $y(0) = 4$, $y'(0) = -5$.

19. Find an integrating factor of $xy\,dx + (2x^2 + 3x^2 - 20)dy = 0$.

20. Solve $\frac{dy}{dx} + y \tan x = \cos^3 x$.

21. Determine the constant A such that the equation

$(Ax^2y + 2y^2)\,dx + (x^3 + 4xy)\,dy = 0$ is exact.

22. Solve $y\cos x dx + 3\sin x\, dy = 0$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions.

23. Given that $y = x$ is a solution of $(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, find a linearly independent solution by reducing the order.

24. Solve $(y^2 + yx)dx + x^2dy = 0$.

25. Solve the differential equation $y'' - 10y' + 25y = 30x + 3$ by undertermined coefficients.

26. Solve $x^2y'' + xy' + y = 0$ subject to $y(1) = 1,\ y'(1) = 2$.

27. Make the following equation exact and hence solve

$(xy^3 + y)dx + (x^2y^2 + x + y^4)dy = 0$

28. Solve $(D^2 + 4)y = 3\sin 2x$.

29. Solve $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$.

30. Solve $x\frac{dy}{dx} + (3x+1)y = e^{-3x}$.

31. Show that for a second order homogeneous linear differential equation, any linear combination of 2 solutions on an open interval I is again a solution of the differential equation on I.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

32. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x.$$

33. Solve $\frac{d^2y}{dx^2} + y = \tan x$ using the method of variation of parameters.

34. (a) Find the orthogonal trajectories of confocal parabolas $y^2 = 4(x+a)$.

(b) Solve $(D^2 - 10D + 25)y = 0$.

35. (a) Solve $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$.

(b) Solve the Bemoulli's equation : $\frac{dy}{dx} + y = xy^3$.

(2 × 15 = 30 Marks)